

THE RHIND MATHEMATICAL PAPYRUS AS A HISTORICAL DOCUMENT

von

Anthony S p a l i n g e r

Traditionally, Egyptian documents relating to the exact sciences have received less attention than historical, economic, or literary texts owing to their specialized nature. That is to say, except for the intrinsic worth of the contents (mathematics, astronomy, medicine), most modern scholars have preferred to sidestep them, leaving in-depth studies to the few specialists in the field. The famous Rhind Mathematical Papyrus is no exception. We possess Peet's admirable edition of 1923 as well as the later compendium of Chase-Bull-Manning and now a recent overview by Robins-Shute; however, except for some specialized studies, Rhind is for the most part overlooked as a document in and of itself¹. True, there have been a number of very useful studies on certain mathematical problems contained in this composition (and in its relative, the Moscow Mathematical Papyrus²), but it can be fairly stated that the short description of the attack on Avaris located on the verso has preoccupied the pens of scholars more than most of the

-
- ¹ An up-to-date bibliography relating to the Rhind Mathematical Papyrus (BM 10057 and 10058) will be found in Robins-Shute, *The Rhind Mathematical Papyrus*, London 1987, 60. The standard text is Peet, *The Rhind Mathematical Papyrus*, London 1923, (hereby abbreviated Peet, RMP) which, with Gunn's review in JEA 12, 1926, 123-37, provides the best discussion available. Chase-Bull-Manning, in the two-volume *The Rhind Mathematical Papyrus*, Oberlin 1927/29, is useful only for the photographs and minor comments on the text. Finally, the earlier comments of Griffith, in: PSBA 13, 1891, 328-32 (setup, inks employed, leaves, etc.), 14, 1891, 26-31, 16, 1894, 164-73, 201-08, and 230-48, are highly informative and in many ways unrivaled.- General remarks and overviews of Egyptian mathematical works may be found in: Gillingham, *Mathematics in the Time of the Pharaohs*, Cambridge 1972, passim and Von der Waerden, *Science Awakening I*, Dordrecht 1975, Chapter I, passim.
- ² For the Moscow Mathematical Papyrus: Struve, *Mathematischer Papyrus des Staatlichen Museums der Schönen Künste in Moskau*, Berlin 1930, with the review by Peet, in: JEA 17, 1931, 154-60; cf. also Nims, in: JEA 44, 1958, 56-65 for the bread and beer calculations (*bš3/bnr*) with SAK 15, 1988. In JEA 15, 1929, 171 note 2, Gunn and Peet briefly note the probable original vertical layout of the exemplar of the Moscow Papyrus.

mathematical problems³. Although the purpose of this study is to present a careful examination of the development of the text - for example, the interrelationships among and between the various problems and their precise position within the text - one healthy by-product of this presentation may be to reawaken an interest in similar papyri, too often ignored in the bulk of Egyptological research.

An in-depth analysis of the organization of the Rhind Mathematical Papyrus provides a self-contained study which leads to useful results concerning its organization, composition and internal historical development⁴. The volume of Robins-Shute has the advantage of providing for the first time color photographs of the original (the BM 10057 and 10058 plus the fragments in the Brooklyn Museum), and although reduced in size, these are more than clear enough for us to proceed with this study. Hitherto, one had to depend upon the black and white photographs of Chase-Bull-Manning or else the earlier 1898 facsimile of Budge. Although the plates in Robins-Shute are small, the clarity of the photographs is more than sufficient for this analysis and one must keep in mind that the handwriting is clear and beautiful, as befits a master reference work probably originally intended as a teaching manual⁵.

³ In particular, note Vandersleyen, *Les Guerres d'Amosis*, Brussels 1971, 35-40 with Helck, in: *GM* 19, 1976, 33-34. A brief discussion will be found in Redford, *History and Chronology of the Eighteenth Dynasty of Egypt*, Toronto 1967, 44 with his *Pharaonic King-Lists, Annals and Day Books*, Mississauga 1986, 151 note 109. In his very useful list of temple libraries on pages 215-23 Rhind is not covered (nor is any other mathematical work). See the related study of Burkard, in: *Bibliothek* 4, 1980, 79-115. - Related studies may be found in Peet, in: *JEA* 9, 1923, 91-95; P. Berlin 6619 - Schack-Schackenberg, in: *ZÄS* 38, 1900, 135-40 and 40, 1902, 65-66 with corrections by Nims, in: *JEA* 44, 1958, 59 and Berlev, in: *BiOr* 28, 1971, 325, referring to p.140 of Schack-Schackenberg, Pl.IV 2, line 1; and P. BM 10250 - Glanville, in: *JEA* 13, 1927, 232-39.

⁴ Despite the title of the study by Gillings, in: *Archives for History of Exact Sciences* 12, 1974, 291-98 ("The Recto of the Rhind Mathematical Papyrus. How did the Scribe prepare it?"), the article ignores the layout of Rhind.

⁵ In general, Griffith, in: *PSBA* 13, 1891, 329-31. The clarity of the photographs in Robins-Shute is still not sufficient to determine exactly where the breaks occur between adjacent leaves. Černý, *Paper and Books in Ancient Egypt*, London 1952, 8-10, 15, 21, discusses the sizes of papyri. This work, added to Borchardt's old analysis in: *ZÄS* 27, 1889, 118-22, is of prime importance for our understanding of Middle Kingdom papyri. Note also, Gardiner, *The Wilbour Papyrus II*, Oxford 1948, 5 and note 3 with Janssen, *Commodity Prices from the Ramessid Period*, Leiden

This justly praised papyrus has been a staple in the diet of voracious Egyptologists as well as historians of the science of mathematics since its appearance in the nineteenth century. Beginning students have found their study of hieratic ably honed with the beautiful hand of the scribe of this treatise and advanced scholars interested in Egyptian scientific thought have not ceased to use this papyrus. Significant when it was first drawn up in the Hyksos Period, Rhind's importance is continually revealed by the wealth of studies that have been devoted to its various aspects⁶. By the twentieth century there appeared the *editio princeps* of Peet, justly praised at the time of its publication and ever-indispensable to scholars both for his masterly treatment of the problems as well as the detailed study of Egyptian mathematics. Soon after came the compendium edition of Chase-Bull-Manning, which contains excellent photographs, all of which are of prime importance for any study, palaeographical or otherwise. Now with the publication of Robins-Shute, we have excellent color photographs, all of which are useful to the scholar as well as to the interested layman, and the simple fact that Rhind graces the public galleries of the British Museum overtly reveals its value to us today.

Having had the occasion to turn to this treatise in a recent study, I felt that it would not be out of place to draw up a detailed analysis of the arrangement of the text, to emphasize its internal makeup, and to provide a helpful guide to the papyrus purely from a compositional viewpoint. This, then, is a partial fulfillment of the goal. I have avoided any discussion of the mathematical problems themselves,

1975, 447-48. - With the kind assistance of Dr. Steven Quirke of the British Museum and the support of Dr. V. Davies, Keeper of Egyptian Antiquities, I was able to see and analyze the Rhind Papyrus. The breaks that occur between the sheets are evident on the following plates of the Chase-Bull-Manning edition: Pl. I; III (right); IV (left); VI (right); VII/VIII (the same); X (middle right); XII (right); XIV (extreme right); XV (left: the color demarcation is clear); XVII; XVIII (right); XIX (center); XX (center); and XXI (center).

In the Robins-Shute edition see Pls. 2, 3, 4, 6, 7, 9, 10, 12, 13, 15, and 16. According to Griffith, in: PSBA 13, 1891, 331, the ninth leaf of BM 10057 is 15 1/2" (or 38 3/4 cm.) and had no strip. These comments are correct.

⁶ For the dating: Griffith, in: PSBA 14, 1891, 29-31 and later 14, 1892, 436; Peet, RMP, 1-3; Robins-Shute, 10-11 (diagram from page 16 of Chase-Bull-Manning); and SAK 15, 1988, 261-262.

except, of course, when their location is discussed, errors in the writing occur, possible dating of certain portions comes under review, etc. Simply as a document, Rhind has in my opinion not received its due, and despite the obvious dependence upon Peet, the reader may find it interesting to know that Griffith's earlier studies in many ways are the framework of my survey.

Before proceeding with this detailed study some background information may be of use as an introduction. That Rhind is virtually complete is self-evident. It was composed of fourteen separate sheets of papyrus carefully joined and in its length and width fits the standard size of Middle Kingdom papyri of the first quality⁷. Compare, for example, the Moscow Mathematical papyrus. The latter document is poorly written, contains numerous errors in its problems, and is approximately one fourth of the Middle Kingdom standard in size⁸. In addition, its arrangement is virtually chaotic (and I employ Peet's description), further evidence that it was a student's work and not that of a master scribe or teacher⁹. None of the problems in Rhind but the very last are muddled or incoherent as in the Moscow manuscript. There is little doubt that we must follow Peet's and Struve's comments regarding the second mathematical "treatise": i.e., that it appears to be a student's copy, probably based on a manual akin to Rhind. A further useful comparison may be made between Rhind and the British Leather Roll (BM 10250), first edited by Glanville¹⁰. The latter presents a series of abbreviated statements of problems, not many, and those *skm* examples were in turn based on some type of operating reference work.

A. TITLE

The original arrangement of Rhind included a layout that was carefully planned¹¹. (See Diagram 1, which is taken from Chase-Bull-Manning). To the extreme right of the recto may be found two parallel vertical lines that cut off the text proper from the title¹². On the verso will

⁷ For the present size of Rhind: James, in Robins-Shute, 6 with Griffith, in: PSBA 13, 1891, 328-31; cf. Černý, Paper and Books in Ancient Egypt, 8-9, and Caminos, "Some Comments on the Reuse of Papyrus", in Papyrus: Structure and Usage, ed. Bierbrier, London 1986, 43-50.

⁸ Černý, *ibid.*

⁹ Cf. Peet's comments in: JEA 17, 1931, 154 with Struve, Mathematische Papyrus des Staatl. Museums, 3, 4, 10 and 12.

¹⁰ Glanville, in: JEA 13, 1927, 232-39.

¹¹ Page 16.

¹² On the vertical lines dots can clearly be seen on Pl.1 and 18 in Robins-Shute; cf. Pl.22 and note Griffith, in: PSBA 13, 1891, 329-30.

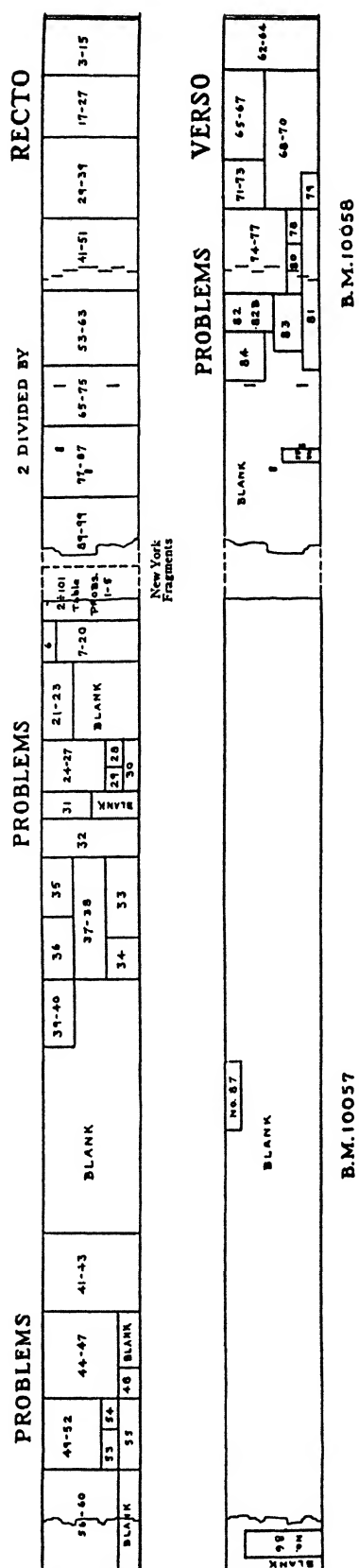


DIAGRAM 1: SCHEMA OF PAPYRUS RHIND
(taken from Chase-Bull-Manning)

be found an identical series of lines at virtually the same location from the edge. On the recto the title of the work plus introductory bibliographic comments are located before those parallel lines, but enough space has been given by the copyist that there is a reasonable margin between the extreme right-hand edge and the first column of the title¹³. The reader of this document could then open the papyrus without being suddenly faced with the start of the text and moreover, he would immediately be able to see the title of the work (plus useful "historical" details) and thereby ascertain just what he had chosen. I would like to hazard the supposition that Rhind was contained with other similar manuals of a first-class nature in a scribal library, quite possibly a school for advanced accountants and technocrats, if I may use those modern-day terms. The reason for the text on the verso remains more unclear and I will treat that portion in Part D below; it is sufficient to note here that nos. 61 and 61b (I shall be employing Peet's designations) were written to the right of the double vertical lines and that these are to a large extent nothing more than a handy series of references to simple fractions paralleling those in the first part of "Book I". I would argue that the verso was originally laid out like the recto - these two lines being added at the same time as those on the recto. With this done, the scribe then began his introduction.

As Griffith was the first to note, the present papyrus consists of various sheets joined together (each c. 40 cm wide) with the right-hand leaf overlapping the edge of the following one to the left by approximately 1/4 inch in most cases¹⁴. None of the publications specifies where the breaks between sheets occur, although it is relatively easy to find them with the aid of the photographs in Chase-Bull-Manning. When completed, Rhind was composed of sixteen sheets - interestingly, the New Kingdom standard was twenty¹⁵. As expected, the horizontal fibers occur on the recto and the vertical ones on the

¹³ On the opening: Griffith, in: PSBA 14, 1891, 27; Peet, RMP, 33; Robins-Shute, Pl.1; Chase-Bull-Manning, Photograph I; and for the date - Redford, Eighteenth Dynasty 44 and note 89 with Helck, Hist.-Biogr. Texte, Wiesbaden 1975, 56 (no.81).

¹⁴ In general, Griffith, in: PSBA 13, 1891, 329-31 and 16 (1894) 165-66.

¹⁵ For the New Kingdom standard, Černý, Paper and Books, 9; Borchardt, in: ZÄS 27, 1889, 120; and Janssen, Commodity Prices from the Ramessid Period, 447-48.

verso¹⁶. Although hard to locate on the very readable color plates of Robins-Shute, there were dots of ink placed on the blank papyrus to form a grid for the series of horizontal lines that occur through the text in the sections on the recto and verso immediately to the left of the two vertical lines. Griffith observed that these dots could mark a "page" (in our sense) but more often were placed at the leaf divisions¹⁷. Clearly, the latter were first drawn and then the ink spots added preparatory to the ruling. Again, we have to thank Griffith for his perspicacity when he observes that the dots are red for the most part: only on the verso of P. 10.058 are they black, and this point will not go unremarked in our discussion in Part D below. Another indication of the divergence between the two sides is the presence of black dashes on the two vertical lines of the verso: these were employed for the delineation of the horizontal register lines¹⁸. On the recto red ink (as expected) is employed at the second vertical line for the marking off of the horizontal lines. As for the latter, there are seven on each side, thereby dividing the text into six portions. Since they cross the leaves, it is self-evident that they were added after the roll was made. Many of the problems fit into a horizontal section and although some overrun their section, this is clearly due to the length of that portion. It is best not to regard the lines as mere guides for a correctly justified composition but rather as prepared horizontal sections into which the various problems would ultimately be written. Of course, this does not apply to the opening series of fractional divisions (see Part B below), but as the manual comprises far more than these handy tables, this divergence from the norm poses little difficulty. Finally, note that the problems could overrun the leaf divisions but that an attempt was made to restrict two "pages" to one leaf¹⁹.

The text proper is c. 30,25 cm in height. The first horizontal lines form a space a bit under 3,35 cm high whereas the remaining five are all c. 5,4 cm. It may have been that the first horizontal portion was

¹⁶ Caminos, "Some Comments on the Reuse of Papyrus", 43, wisely eschews entering the recto-verso controversy, but provides the latest literature in note 3.

¹⁷ PSBA 13, 1891, 329; good examples may be seen in Robins-Shute, Plates 1 and 3.

¹⁸ Griffith, *ibid.*

¹⁹ *Ibid.*, 329-31; this is best seen on Robins-Shute, Pl. 22.

originally intended as some type of heading, although the layout of the composition as well as its intent (i.e., division into separate problems) would tend to negate this supposition. One might even hazard the speculation that a greater number of horizontal registers was first planned, and then reduced to a lesser number. However, here as well we enter the realm of pure guesswork. In some cases a problem overruns a horizontal section and thereby moves down into the next register; other cases show parts of the margin being employed. These anomalous examples will be covered below when the analysis of the composition of the text is covered.

The main portion of the verso containing additional mathematical problems (= problems 62-84) is not as regular as the recto. Leaving a more careful analysis until later in this paper, I only wish to state at this point that the guidelines follow the same pattern as on the recto. However, after the mathematical problems are finished (with no. 84), the guidelines run on for a bit at the top and bottom while the rest appear to cease; eventually even the borders end and by the time the verso of P.B.M. 10057 is reached, the papyrus was originally left blank²⁰.

The title itself is written with red ink in two vertical columns; the bibliographic details employ the normal black ink but also follow the columnar pattern. It is correct that a vertical arrangement was abandoned for hieratic documents by the close of Dynasty 12, and that the problems themselves are written in a linear fashion (from top to bottom). However, it must be kept in mind that this setup easily fills the available space with little overlap and that a horizontal arrangement of at least the title would have required a series of many eye-movements from line to line. In this case, the red-inked title (*tp-ḥsb n ḥ3t rh ntt nbt snkt ... [s]št3t nbt m ḥt*) stands out clearly in the blank of the extreme right. Note that the additional words that present the background of the manuscript are written in black. There also appears the date of the text, the date of the original from which it was copied, and the name of the scribe. It is significant that these details were added at this point somewhat later: they follow

²⁰ Möller, *Paläographie I*, 7-8; Griffith, in: *PSBA* 13, 1891, 331; Simpson, "Papyri of the Middle Kingdom", in: *Textes et langues de l'Égypte Pharaonique II*, Cairo 1972, 63-72, covers the *Moscow Mathematical Papyrus* (p. 68) but not Rhind, as it is outside his purview (Middle Kingdom papyri).

immediately in column 1 underneath the $\check{s}t3t\ nbt$ and proceed into column 3; the title itself occupies most of the first column (up to nbt) and concludes with $m\ ht$ in the second; nothing is located below the ht . This additional notation carries on down further than the baseline in the text located to the left of the horizontal lines for the simple expedient of completing the notation within the available space.

The details of date of copying, etc., provide us with the well-known background to the composition: i.e., year 33 of the Hyksos ruler Apophis. An interesting verbal formation, $\dot{i}w\ \dot{i}st\ grt\ sphr.n.tw\ \check{s}fdw\ pn$, opens this portion of the text and there then occurs the troublesome reference to the original exemplar from which Rhind was copied. Peet's interpretation was that Rhind depends upon a lost copy from the reign of Amenemhet III ($Ny-m3^Ct-R^C$) of Dynasty 12²¹. Earlier, Griffith, in a more cautious fashion, had opted for this restoration but was more emphatic in rejecting a Middle Kingdom date for the problems, owing to the presence of the quadruple hekat measure²². A recent reinterpretation by myself concerning this section of the opening casts doubt upon Peet's reading of the Pharaoh's name in the lacunae since the writing in columns had been overlooked²³. Additional support for this will be gathered in more detail later in this study. Suffice it to say that the restored name $[Ny]-m3^Ct-[R^C]$ does not fit the space unless perhaps only $nsw\ [Ny]-m3^Ct-[R^C]$ be argued, and even then the situation remains moot and the lacunae can be interpreted differently. Finally, observe that the black-inked additions extend the original length of the first column down further than the red title, and in the last column the very bottom of the scroll has been reached. In fact, the beginning of the bibliographic details, $\dot{i}w\ \dot{i}st\ grt$, skips over the red-inked conclusion of the title in column two by continuing on in the third with $sphr.n.tw$. . ., as if the intent was to keep part of the title still separate. From these indications I feel that the biblio-

²¹ RMP, 3 and 33. For the verbal formation referred to in the last sentence: Polotsky, in: RdE 11, 1957, 111 and "Egyptian Tenses", in: Proceedings of the Israel Academy of Sciences and the Humanities 2.5, 1965, 9-10 and note 22 (= $\dot{i}s\ \dot{i}st\ grt + sdm.n.tw.f$).

²² Griffith, in: PSBA 14, 1891, 27, 29-31 with 16, 1894, 164; his qualifications can be found in 14, 1892, 436 and the note.

²³ SAK 15, 1988. If it is argued that only a nsw instead of $nsw\ bity$ is to be read in the lacuna, the entire question must be placed to one side: i.e., we arrive at an argument *ex silentio*. Some may prefer this position.

graphic details were added at the conclusion of the work, as befits the scribe who drew up the text. After all, the typical conclusions to literary texts normally included the name of the copyist and were clearly written after the text was finished.

B. BOOK I

Peet's division of the text into "Books", which was only slightly modified from Griffith, has justification not only from our present-day criteria but also from the layout of the text. Basically, the first portion of Rhind begins with the divisions of 2 and then runs into a series of general mathematical problems, which are nevertheless concrete and representative of the practical day-to-day situations that were expected of the graduating apprentice. The large break between Book I and Book II as well as the switch from these problems to more complex ones covering measurements (volumes of pyramids, etc.) present even more conclusive support for this separation. If his rubrics of "Arithmetic" for Book I and "Mensuration" for Book II are not too severely criticized, then the emphasis of the two parts can be even more clearly seen²⁴.

1. FRACTIONAL DIVISIONS

The fractional divisions of 2 by odd integers ranging from 3 up to 101 were meant as a handy table for future calculation. Since each fraction $2/n$ was worked out, it had a practical purpose as well: namely, to teach the student the correct method of determining these difficult (to the Ancient Egyptians) calculations.

The first block (see Diagram 1) covers 2 divided by 3, 5, etc. up to 15. The *vertical* rubric, *nꜥs* (in red ink), is only fully written at this point (to the right of the various calculations). The use of this verb emphasizes for us the oral nature of Rhind's original purpose: the teacher read out the problem and the student worked on it. Of course "divide" is what the context demands; however, the core meaning of the verb (probably in the imperative) should not be disregarded. Slips of a trivial nature occur in all of the fractional nota-

²⁴ Peet, RMP, 4-5 and 80; he follows Griffith, in: PSBA 16, 1894, 165 with 13, 1891, 332.

tions and they have been remarked upon by Peet²⁵. By and large they in the nature of omissions, such as the failure to place the calculating dash on the line to be added or else omission of the dot from the fractions; the more serious errors will be discussed in passing. Adjacent to each statement is the "working out", located to the left of the problem and not below. The word *sšmt* is only written in the first line and as may be expected, like *nīs* which is written only once, it serves as a heading for the problems underneath the first one. As the computation of 2 divided by 3 was trivial, that connected with 2 divided by 5 is located immediately to the left and occupies the first three lines; the next series of fractions commencing with 2/17 are located to its left, whereas 2 divided by 7 up to 15 are correctly placed below. Significantly, the break between the first leaf and the second occurs just before the "working out" of 2/5 after the word "*sšmt*", and that word is closely pressed against the opening "result line" of 2/5 with the final black 1/3 barely separate from the hieratic owl; it must have been added immediately after²⁶. As for the horizontal guidelines, the examples are well located within them: 1/3 and 1/5 occupy the first section and each of the remaining are located within a single register which encompasses the problem title and solution in the first line with the method of solution (*sšmt*) in the following ones.

Red ink is consistently employed within these calculations to highlight key numbers or operations. For example, the answer for 2 divided by 3 is rubricised; likewise, 2/7, 1/4 and 1/28 are in red. Two additional red diagonals may be seen to the left of *sšmt* and after the numeral 5 in the first line; they are the normal calculation tics.

The next block of the manuscript encompasses the fractions 2/17 to 2/27 and again, six calculations are covered²⁷. The operational commands *nīs sšmt*, located separately to the right of the arithmetic, are rubricised but the second is this time placed under the first and

²⁵ They may be noted in the translation pages (in the footnotes) to Chase-Bull-Manning as well as in Peet's commentaries (see page 38 at this point) to each problem; the former work is more detailed. Only when such mistakes are of significance will I specifically refer to their comments; cf. Peet, RMP, 38.

²⁶ Chase-Bull-Manning, Photograph I (top); Robins-Shute, Pls. 1-2.

²⁷ I employ the word "block" instead of "page" owing to its more concrete visualization; cf. Diagram 1. For the text: Chase-Bull-Manning, Photograph II; Robins-Shute, Pls. 2-3.

the associated figures to the left are effectively highlighted. Likewise in red ink are the two strokes for 2, reiterating the purpose of the problem. It has been noted that "in most of the multiplications carried through for the purpose of obtaining fractional multipliers that will produce given fractions the partial products are written without dots"; i.e., the numbers are presented as if they were integers instead of fractions²⁸. This error, or slip, is of little consequence, as Peet noted: the reciprocal methods of multiplication and division were present in the scribe's mind. As the denominators increase in size, so do the calculations, and the words *dmd* and *d3t* appear consistently from now on.

Within the group of 2/29 to 2/39 (again six cases to fill the six registers) the third leaf is reached. This subsection is longer than the preceding two, owing to the length of calculations involved. For 2/35 a helpful analysis of sixths occurs - 6 (red) followed by 7 and 5, the latter two indicating that 7 sixths are in 1 1/6 and 5 in 2/3 + 1/6. The first 6 is not a fraction; indeed, it could never be so. Rather, here we see the concrete nature of the Egyptian conception of fractions: i.e., as parts of a larger group but also as whole units. The Egyptian's conception of fractions is not like ours. A sixth is concrete and forms a unity in and of itself. Six of them form another unity in which there are six parts. The sign for 2/3 provides a good case in point. The hieroglyphic shows one unity twice as large as another; together, they form a unity. Moreover, the larger part is twice the smaller; hence, the greater unity is three times the smallest.

The arrangement of each of these blocks of fractions is interesting to see. The standard setup was to first write the *n2s* 2 with *s3mt* underneath in red; the *n2s* was abbreviated. To the left was placed *hnt* and then further left, the number to be worked on. The solution was presented on the same line, with the various subfractions indicated. Below was the working out in the standard Egyptian method of division or multiplication. In the third block (2/41 to 2/51) we meet for the first time the word *gm*, "find", with 2/43. I follow Peet in his supposition that the problem depended upon the use of other tables

²⁸ Chase-Bull-Manning on page of translation "2 Divided by 17". For the discussion of "2/3" in the following paragraph, cf. Clère, in: *ArOr* 20, 1952, 629-41.

not included in Rhind, although in some cases (such as this one with 43/42 and the 53/30 later on) it may be argued that the "paper-work" in obtaining the result has been omitted²⁹.

The regularity of each block of fractions quickly impresses one's eye. Each series of fractional divisions is easy to read. As a result, the red and black inks stand out in full clarity. The spaces between the hieratic signs are uniformly wide enough for quick perusal. In similar manner, the computations of the preceding block of six fractions never intrude upon the next block, each grouping maintaining an independence from the one preceding and the one following. An attempt has also been made to present the red figures in the first (result) line in about the same places as those below; from top to bottom one can observe the regularity of the system: by and large the rubricised figures can be found in roughly the same places as those in the first line of the papyrus. Space itself was not considered to be at a premium, even though these fractional problems formed the basis of the mathematical treatise rather than the meat of the work, and were therefore preparatory to it. It is true that the scribe normally began a fresh "page" (my "block") "at the point where the longest line of the preceding page ends"³⁰. However, I do not see any severe cramping in the problems so far covered, except possibly that occurring in Peet's fragment 19 (the end of 2/89), and even there it is minimal³¹.

2. THE FIRST SERIES OF PROBLEMS

The final block consists of only one $2/n$, namely 2 divided by 101 and a new series of problems³². There is no overt separation between the close of the first table, which occupies the first register, and the following series of calculations (1 to 9 divided by 10). For Peet, Book I (his "Arithmetic") began here with the double columns of $1/10$ to $9/10$. I feel that this is a reasonable assumption, especially as the latter calculations are connected with the first six arithmetic problems of Rhind that immediately follow (division of 1,2,6,7,8,9

²⁹ Peet, RMP, 42-43, especially his comments on the top of page 43; Chase-Bull-Manning on page of translation "2 Divided by 43" with Pl.13 and Photograph VIII.

³⁰ Cf. Griffith, in: PSBA 13, 1891, 31 on "pages" to "leaves".

³¹ Chase-Bull-Manning, Pl.30 and Photographs VIII-IX; Robins-Shute, Pl.6.

³² Chase-Bull-Manning, Photographs IX-X; Robins-Shute, Pls. 8-9.

loaves among ten men). And in the latter we meet for the first time the phrase *irt mī hpr* (in black), an arithmetical heading that occurs frequently in the later problems. Of greater importance is the fact that we have now entered the realm of practical problems that by their nature depended upon fractional calculations such as those first drawn up. Nevertheless, there is no division overtly noted; no blank occurs as is common later on (e.g., between problems 40 and 41).

One might query at this point the source or sources of Rhind. Did the original exemplar contain the opening table as well as the subsequent problems or, to complicate the case further, was that treatise itself derived from various unknown works now lost? That this is not idle speculation can be seen from Griffith's remarks concerning the grain measures employed³³. He stressed the presence of the quadruple hekat in this papyrus, a measure which was unknown to him as a standard in the Middle Kingdom. We can now refine his argument in light of subsequent data³⁴. In Rhind the quadruple hekat occurs in Books II and III but not in Book I, in which only the single hekat occurs (problems 35 to 38). From data more recent than Griffith had at his disposal, the same situation appears to have been the norm: in the Middle Kingdom (Dynasty 12) only the single and double hekat have been found; one has to wait for Rhind to note the presence of its four-fold companion. Since a large blank separates Book I and the table preceding from Book II, and the latter commences immediately on a new leaf (the twelfth), the divisional intent seems emphatically evident. Can we therefore assume that Book I represents the copy mentioned at the beginning, and Book II (as well as the problems on the verso) another source (or Sources)? After all, was so large an empty space necessary? Whether or not one wishes to include the opening fractions (2/3 etc.) with problems 1 to 40 in the original copy, I feel that the break after problem 40 is large enough to warrant this query. Moreover, when it is remarked that the quadruple hekat occurs in Book II and even a "great quadruple hekat" in Book III (problem 69) as well as a triple hekat, the situation becomes even more worthy of investigation. If only for these reasons, I am of the belief that the sources of Book II (and III, but this needs more clarification: see Parts C and D below) was either different from that of Book I or else a reworked

³³ See note 22 above.

³⁴ SAK 15, 1988.

series of problems having their origins in the copy that scribe Ahmose employed.

At this point Peet queried the situation wherein the opening divisions present a complete series but the following bread divisions, which are connected to them, present only six cases³⁵. The arrangement of the text appears to me to reveal a possible solution. With the division of 2 over 101 completed in the first register, the scribe then proceeded with his $1/10$, $2/10$ up to $9/10$ in the second. All of the last were in black ink and served as a handy table for the bread calculations. To the left of these and still within the second register, was placed problem one: divide 1 loaf among 10 men. (This case Gunn considered to be the paradigm)³⁶. Quite possibly for reasons of space, our scribe avoided giving all of the bread examples but still ran over the block and so had to finish up with 6 loaves divided among 10 men in the first register of the next block. Even then, this meant that an even division of loaf problems into one block (or two) could not be attempted unless there was a further reduction of these problems. It is also significant that the scribe placed his opening arithmetical computation for one loaf divided among 10 men to the left and therefore next to the $1/10$, $2/10$, etc. chart. Furthermore, he justified problems 2-5 in the lower registers to problem 1 by placing the commencement of each line directly under the rubricised *tp* (of *tp n psš*). Despite this, there is some evidence of fraction crowding in this series of problems, mainly owing to the height of a fraction³⁷. Note that except for the opening of *tp n psš* (red ink) *t n s 10*, the subsequent headings read *ḫrt t X n* (red) *s 10* (*X* is an integer), thereby indicating the connection of the five latter cases with the opening example.

Immediately under the simple bread division of 9 loaves among 10 men are a series of "completion" or *skm* examples. Peet has labeled them the "First Group of Completions" and they occupy the remaining seven eighths of the block (see Diagram 1)³⁸. Significantly, these elementary

³⁵ See note 32 above; add Peet, RMP, 50-51.

³⁶ See note 32.

³⁷ Robins-Shute, Pl.9 (bottom right register).

³⁸ Peet, RMP, 53-54 (nos. 7-20); cf. Griffith, in: PSBA 16, 1894, 232 with Chase-Bull-Manning, Photograph X and Robins-Shute, Pl.9.

problems are not separated from those preceding by any blank. Instead, they have been likened to "experimental methods" wherein complex additions and multiplications of fractions take place. None of them are connected to any situation in real life; rather, problems 7 to 20 are a series of handy arithmetic calculations of an advanced type that would be presented to students to solve. It is significant that the mathematical leather roll of the British Museum concerns itself mainly with such *skm* examples, and Glanville stressed this by observing that the scribe-student of this roll probably was given them to solve from a more lengthy list such as that contained in Rhind³⁹.

The order of these separate problems is as follows:⁴⁰

REGISTER 2. No.7 - includes the heading *tp n skmt* in red; every other line is rubricised, as they note the use of a common denominator (28 in this case).

REGISTER 3. Nos. 8, 9, 7b and 10. Four problems occupy each register as the standard from here on (except for the last). Problem 7b is a repetition of no. 7. There is an error in no. 9 (not corrected) similar to those in problems 10-12 and 14⁴¹. The rubricised lines follow the standard of problem 7. No reason has been given for the repetition of the last and it may have been that problem 7 was the paradigm and 7b, its duplicate, included among the series of problems to be solved. Note that a very good effort has been made to align each line in the first right-hand problem (no. 8 in this case) with those to the left⁴².

REGISTER 4. Problems 11-14. Here, corrections (lighter ink) can be seen in problems 11 and 12. The lines are neatly aligned to one another but the red figures tend to be smaller and cramped, especially in problem 13, and it might be surmised that at least some of the red lines were added later, if only owing to their constricted state (cf. those in problems 13 and 14). Others, however, are of normal height and width. Perhaps when the later corrections were added, the

³⁹ Glanville, in: JEA 13, 1927, 234 and 238 on the *skm*-examples; cf. Gunn, in: JEA 12, 1926, 129, on these problems and reasons for the absence of certain divisions: e.g., division by 5 may have been self-evident. On the use of *tp n*: Vernus, in: RdE 33, 1981, 111 note b.

⁴⁰ Most clearly seen on Robins-Shute, Pl.9.

⁴¹ Peet, RMP, 55.

⁴² Chase-Bull-Manning, Photograph X and Pl.40 with Robins-Shute, Pl.9.

scribe wished to add some ancillary notations concerning the common multipliers employed in his calculations.

REGISTER 5. Problems 15-19. The red figures are rarely encountered in these problems. No. 15 has a note in the margin to the right - "defective", and it is in the lighter black ink once more⁴³.

REGISTER 6. Problem 20. There is a rather long space to the left as the last "completion" is not lengthy. The red figures are again cramped.

Problems 21-23 occupy the first two registers of the following block⁴⁴. They are in turn followed by a large blank immediately below, cutting them off (and those preceding) from the next series of mathematical problems. Such a division is partially apparent just after problem 20. To the left of the one column of figures is an empty space, and I believe that Peet was correct in seeing a different series commencing with problem 21. However, in this case the scribe decided to run them from the top of the next block instead of placing them immediately to the left of no. 20. It is difficult to determine whether the original layout was intended to follow the final arrangement: i.e., the radical separation of nos. 21-23 from those preceding. However, the liberal use of blanks on Rhind appears to support this contention, especially when it is remembered that no. 20 could not be placed to the left of no. 19 as that would have meant running over the left-hand margin of its block.

Problem 21 itself occupies all of the first register and is divided into three distinct subsections. The first is (in red) $\underline{dd} \ n.k \ skm \ m$ (line 1) followed by the solution. To the left is the $w3h \ tp$ subsection in a separate column: $w3h \ tp \ m \dots$. The third column has $tp \ n \ syty$ upon which Peet cast some doubt, and the last row of fractions is cramped into the register. Certainly the fourth column, $ky \ 1/5 \ 1/10 \ m \ w3h$, does not belong to the problem, as Peet observed, and the original may very well have been faulty. However, Peet also noted that the connection of the $1/5 (+) 1/10 \ m \ w3h$ belongs to problem 35 as

⁴³ For the reading: Gunn, in: JEA 12, 1926, 129; cf. Peet, RMP, 56 and possibly Gardiner, The Wilbour Papyrus II, 185; cf. Meeks, Alex I 14, II 17 (- especially Faulkner's comments on page 66 note 13 in the Coffin Texts I, Warminster 1973), and Lesko, A Late Egyptian Dictionary I, Berkeley 1982, 17. The notation is best seen in Robins-Shute, Pl. 9 ($\dot{t}3t$ to the right and outside of the block).

⁴⁴ Chase-Bull-Manning, Photograph XI with Robins-Shute, Pls. 9-10; cf. Peet's comments, RMP 58.

well⁴⁵. Can it be the case that the original exemplar had problem 21 immediately to the right of no. 35 (both, perhaps not coincidentally, occupy the first register in Rhind), and that contamination has taken place? And as the phrase *tp n syty*, which makes no real sense in problem 21, occurs once more in nos. 32-38, I feel that this supposition has some merit: viz., the scribe's eye may have slipped too far to the left and a bit of problem 35 crept into no. 21.

Problems 22 and 23 occupy the next register and then there is the enormous blank below. The second is cramped owing to the desire of the copyist to limit his use of this block to only two registers, and quite small signs in red ink are again employed for the common divisors. Whereas problem 21 is more stretched out to fit into its four separate columns, 22 uses four lines instead of three and therefore has only three columns; 23, very tightly contained in register two (and even overrunning it a bit), has two columns. The last employs red ink for the calculations but 22 does not.

With the "completion" series at an end, the scribe then began another block on the next separate leaf. In fact, 16-23 were written on one leaf, thereby fulfilling Griffith's remarks that generally two "pages" (my "blocks") occupy one leaf in Rhind⁴⁶. Peet labels nos. 21-23 as a "Second Group of Completions" and I believe that this separation is correct. These three are sufficiently different from the preceding to be given another heading. However, they do join up in general plan and operation with problems 6-20: after all, they include the word *skm* as well. Nevertheless, they are so different from the trial-and-error solutions to linear equations which follow that the large break below no. 23 has great meaning. In "Book I" this is the only blank that occurs. The only other unused space that comes to mind is that occupying most of the space in the lowest register in the preceding block to the left of problem 20. In both cases the switch in approach is noted by a physical separation, and the greater one (between 23 and 24) is likewise more prominent with respect to topic change. Such free use of space reminds us of the lack of constraint evident in this papyrus: despite the intrinsic importance of the document, and cost of material, Rhind appears to have been deliberately under-utilized

⁴⁵ Peet, *RMP*, 58-59 (versus Griffith, in: *PSBA* 16, 1894, 233); and note Robins-Shute, *Pl. 21*.

⁴⁶ *PSBA* 13, 1891, 329-30.

in terms of space. Certainly, the blanks (there are more to be noted) bear ample evidence for this interpretation.

Peet has divided the next group of problems (nos. 24-38) into a separate section, that of trial-and-error equations of one unknown⁴⁷. Whereas it is true that they all can be subsumed together, there are enough inherent differences among them to warrant separate subdivisions. Nos. 24-25 are the simplest, opening with the title "A quantity ($^c h^c$) whose Xth part is added to it becomes Y" (in red ink). The first, located in the first register, divides the preliminary calculations (right side) from the "proof" (left side) and in the latter the scribe, originally omitting the $^c h^c$ ("The quantity is"), has placed this word high up and a bit over the top margin⁴⁸. No. 25, occupying the second register, rubricises the second $^c h^c$ but not the following number. In the third register is no. 26, a problem; although identical in structure to the preceding two, it is more wordy. Here, the first line is rather lengthly and clearly ran too long, as the next block of problems is virtually inseparable from it. Note that the $^c h^c$ plus number in the "proof" section are both written in red. By the time no. 27 is reached some errors have begun to enter, quite possibly a result of a faulty original. This one parallels the opening two in its simplicity and I feel that the Chase-Bull-Manning edition was correct in linking the opening four problems together⁴⁹. The final line (the fifth) has overrun the register so that problem 28 begins lower than expected within its register (the fifth)⁵⁰. The $^c h^c$ + figure are in red as is the final notation "total 21" (left, second line). Note that an error can be found where "15" is written instead of "14". As the mistake ought not to be due to a fault in hieratic (the units "4" and "5" are distinct), I would place the onus on the copyist, who was perhaps remembering the use of the trial number 5.

It is with problems 28 and 29 (fifth register) that difficulties set in. The first lacks the "proof" whereas the second contains only the second half of a problem⁵¹. Can we assume that the original was broken

⁴⁷ Peet, RMP, 60; Chase-Bull-Manning, Photographs XII-XV; Robins-Shute, Pls. 10-13.

⁴⁸ In particular, see Robins-Shute, Pl. 11 (top) with Chase-Bull-Manning, Photograph XII and problem 24, Pl. 47.

⁴⁹ Peet, RMP, 63.

⁵⁰ Chase-Bull-Manning, Photograph XII; Robins-Shute, Pls. 10-11.

⁵¹ Peet, RMP, 63-65, provides useful comments; cf. Chase-Bull-Manning, Photograph XII and Pls. 50-51. For problem 31: Chase-Bull-Manning, Photograph XIV; Robins-Shute, Pls. 12-13 (register 4).

away at this point, the lacuna occurring between the two problems? Or, perhaps, did our copyist's eyes wander a bit? To choose between the two is difficult, and I can only note that, as the arithmetic has become more detailed, the errors have increased in roughly equal proportion.

No. 30, occupying the final register of this block, presents a different heading, although the problem is of the same nature: "If a scribe says to you" (in red). The rubricising continues with the internal *dmḏ p3 ḥḥ ḏḏ sw* ("Total: the quantity that says it"). The "proof" section is not separated from the opening by any columnar arrangement as, for example, in no. 24. The final note, "total 10", is once more in red.

The following block is more condensed, vertically, than before. There is only one problem represented, no. 31, and it runs into register three but ends there, occupying just one line of that section⁵². In fact, the calculations are horizontal in position and not divided into two separate columns as, for example, in nos. 24-25. This is due both to the complexity of the problem as well as to the overt desire of the scribe to limit the amount of horizontal space occupied by the problem. Yet despite the available room within registers 4-6, the copyist has had to add a further calculation in register 4 a f t e r problem 38! It is self-evident that this was done later, quite possibly when this portion (i.e., Book I) of the manuscript was re-read for checking purposes, or possibly after the work was completed. Significantly, these three lines of additions are located to the right of a large blank separating the problems of Book II from those preceding, and on the basis of this later addition, I feel that the present scholarly division of Rhind into Books I and II has clear justification from the arrangement of the ancient scribe.

Returning to the problem at hand, note the error of 99 instead of 97, a bad one that cannot be blamed on the hieratic original. In fact, problem 31 bears the traces of a cramped and difficult-to-read copy, probably not justly representing the original. Perhaps for this reason Peet has called the layout "extraordinary"⁵³. However, his attempted

⁵² Chase-Bull-Manning, Photograph XIII; Robins-Shute, Pl. 11; and Peet, RMP 66-67.

⁵³ RMP, 66.

explanation of the interchanges of the various steps makes perfect sense. According to Peet, Step 1 (opening) was originally located to the right of the second and third. The copyist misplaced Step 4 (final one) underneath the first, perhaps due to a slip of the eye. Step 2 then occupies its present place at the end of problem 31 simply because its original location in another column was to the left of Step 1: i.e., the copyist then moved his eye upwards to the left, the original order having been carelessly misread. As for Step 3, an intermediate one, it was left off, only to turn up on Rhind after problem 38⁵⁴. I suspect that that step was in the original so far to the left that it could only enter into the copyist's apprehension at the point where the next block on the original was reached.

Problem 32 then follows, and occupies a complete block of six registers. Why the scribe did not begin it below the preceding is difficult to answer. The query is even more provocative if it is noted that no. 32 runs over the final base margin line. Nevertheless, at this point the neat arrangement hitherto prevailing breaks down. On Diagram 1, taken from the Chase-Manning-Bull publication, only part of the skewed setup can be seen. In fact, this schematic outline is too facile, as the sections are not in fact set up in so simple a fashion. Excluding the situation wherein a portion of problem 31 appears to the left of no. 38, the following ought to be stressed:

- 1) no. 31 is only written in the first four registers and in the last only one line occurs.
- 2) Problem 32 overruns its block at the base margin. From the setup I feel that it was originally composed in separate columns, but for the purposes of the copyist of Rhind, slightly compressed into a one columnar block.
- 3) Problem 33, which by its nature must follow 32, is located in the final two registers immediately to the left of those of problem 32, and it overruns the base margin as well. It is composed of two columns of work and, owing to the length of the final lines of the first (7-13), there is a handy line separating it from the second column of calculations.

⁵⁴ Peet, RMP, 67; Chase-Bull-Manning, Photograph XV; and Robins-Shute, Pls. 12-13.

4) Problem 34, which is parallel to the last, is located immediately to the left. As with the preceding, it is written in the two bottom-most registers and also a bit under the lower margin. It is short and I suspect was originally written in two columns, the second of which now occurs below the first. Once more, the lower margin line is over-run.

5) Problem 35 introduces a more practical version of the preceding. In this case, and those that follow, the mathematical problem concerns hekats (single ones), although the method of operation is identical to those immediately preceding. It, and its partner (no. 36), are located in the first two registers with a moderate space occurring at the base of the second register for no. 35. Note that there is a large space in the first two registers between problems 35 and 32 (to the right). Down further, the latter runs almost up to the beginning of problems 37 (register 3), 38 (register 4), and 33 (registers 5-6). To the left of 35 there is a reasonable spacing between it and no. 36. Note the opening dd sw in no. 35, a phrase that also occurs in problems 30, 36, and 37 (thereby linking all together). Gunn, in his review of Peet, places some emphasis on the use of *sw* in this phrase (dd sw) instead of *st*; probably the use of the masculine dependent pronoun singular indicates the early date of the original⁵⁵.

6) Problem 36 is written to the left of 35 and it, too, occupies two registers. As Peet noted, it originally dealt with the single hekat, although this is nowhere explicitly stated⁵⁶. He further argued that something may have been left off as there was no reduction to the Horus eye fractions of the hekat measure. Unless it is argued that 35 is out of place, I believe it wise to follow Peet's position and consider the probability strong that there is something amiss with the problem. Note that the margins of 36 in the second register continue those of the first, even though there is a lot of space to the right since no. 35 is somewhat short at this point. It is also interesting to note that the separation between 35 and 36 occurs to the right of that between 33 and 34 (lowest two registers). Clearly, the setup lacks the neatly drawn blocks of the preceding problems.

7) Problem 37 occupies the entire third register. It resumes with the

⁵⁵ JEA 12, 1926, 131.

⁵⁶ RMP, 73-74.

portions of the hekat and therefore belongs to the more practical problems with which Rhind is concerned after the preliminary and more abstract ones have been solved.

8) Problem 38 is similar in layout to the preceding and likewise occupies only one register (the fourth). Various columnar divisions can be seen (as with no. 37) and it ends somewhat short in the opening two or so lines. As noted above, part of problem 31 can be found there to the left, clearly added later and in a space that does not disfigure the arrangement in any way.

This entire section can be schematically diagrammed as I have done in Diagram 2. Why such a drastic alteration from an extremely orderly system of blocking should occur is hard to discern. However, the slips that occur are many (e.g., that connected with problem 31 or the briefly written 36) and lead me to suspect that the fault lay more with the scribe than in the original itself. In fact, one might add that the difficulties begin with no. 31, noting as well that for some reason the copyist decided not to use the remaining registers (at least three) under the last line of 31.

If space on the papyrus was not the problem, and I believe that this is eminently the case with Rhind, than we are led to believe that such omissions as occurred in no. 31 - which was, after all, later imperfectly corrected - must be due to the careless or tired eyes of Rhind's scribe. Can it also be the case that he moved from the recto of his original to the verso, and that the originals were oddly set out, leading to the somewhat incoherent arrangement of Rhind at this point? A further possibility may be to envisage the original ending on its recto at problem 34. After all, there is a thematic break of sorts between 34 and 35. If so, the copyist may have decided to extend the bottom two registers by filling them up with nos. 33 and 34. Then he could have turned the original over and later decided to add nos. 35-38 (and 39-40, but the situation is more complex: see the comments below). However, the right-hand margins of problems 35, 37, 38, and 33 are as closely aligned to one another as might reasonably be expected (as are the left-hand margins of 36, 37, 38, and 34), and it seems probable that the lower ones followed the upper in the expected normal order of writing. Hence, I would argue that the scribe simply omitted part of the text, i.e., nos. 33-34, and that he added them

later underneath no. 38. Immediately after, he could have included the remaining portion of problem 31 to the left of 38 at the same time. In other words, a correction occurred in the middle of the work on Rhind rather than after the entire papyrus was finished. Other possibilities come to mind as well, instead of the arrangement on the original as drawn in Diagram 2. In this case one could argue that the scribe moved downwards to the right from no. 32 to no. 35 and then carried his work through to no. 38 only to return to no. 32. Such speculations may be of some use as a possible clue for the odd arrangement of Rhind; nevertheless, they still lack firm support.

The final two problems of Book I are not connected to those preceding. Again, the difficulty arises concerning the thematic breaks in Rhind. At this point, nos. 39-40 present in the first two registers of the following block a calculation of shares of loaves divided unequally among a given number of men⁵⁷. The situations are therefore totally distinct from those of the preceding group, as they deal with proportions rather than equations. Book I ends at this point. It might be argued that these two problems were added later, as the tenth leaf begins just before the right-hand margin of nos. 39-40, and the next writing appears immediately at the beginning of the twelfth leaf: i. e., the break between Book I and Book II took up two (blank) leaves. Certainly, this would have made a neat division between these two major sections of the mathematical treatise, and in light of the disorderly layout of the final problems of Book I, I feel that this supposition is worthy of consideration. Moreover, the large blank below problem 23 acts as an internal break in Book I and it may equally be argued that such a definite hiatus ought to have occurred after the problems dealing with simple linear equations.

Nos. 39-40 are the only ones covering the division of breads by shares or excess (*tnw*), and they are written in the first two registers of the following block⁵⁸. This is not correctly noted in the Chase-Bull-Manning publication but may easily be seen in the Plates to that publication, as well as those of Robins-Shute. Whether or not other similar problems had originally belonged to this series is, of course, unknown.

⁵⁷ RMP, 77-79; Robins-Shute, Pl. 13; Chase-Bull-Manning, Photographs XV-XVII.

⁵⁸ On *tnw*: Peet, RMP, 77; Gunn, in: JEA 12, 1926, 131 (*dwn* and *tnw*); Caminos, LEM, 314 with A Tale of Woe, Oxford 1977, 16; and Alex I 433, II 429.

DIAGRAM 2 : SCHEMA OF PROBLEMS 31 - 38

REGISTER 1		36	35	BLANK	32	31
REGISTER 2		36	35	BLANK	32	31
REGISTER 3		37		32		31
REGISTER 4		31	38		32	31
REGISTER 5		34		33	32	
REGISTER 6		34		33	32	
		34	33	32		

C. BOOK II

The large and definitive break between Books I and II was carefully set up. The mathematical problems that follow the break are quite different from those preceding, and from here on abstract introductory problems of an elementary nature are avoided. Instead, Rhind turns to the situations of volumes (nos. 41-47) and areas (48-55), and then concludes with heights (*mryt*) and batters (*sḫd*) of pyramids and probably a pillar or column (no. 60). In essence, we are presented with concrete cases of architectural problems that the apprentice would have to work with in his forthcoming career, and Peet's title of "Mensuration" is a reasonable heading to Book II.

The entire series of problems (nos. 41-60) are a self-contained unity; they do not occur on the verso. In addition, the internal arrangement is well-organized and lacks any of the incoherence that can be seen at the end of Book II. Equally of note is the presence for the first time of the quadruple hekat. (The term "great quadruple hekat", which occurs on the verso in no. 69, does not occur at this point). Problems 41-47 refer to this measure in connection with the various volumes under consideration; clearly, the concept must relate to storage bins, silos, or granaries. The remaining cases are not concerned with containers but with actual buildings, and so no hekats occur. I still follow Griffith when he observes the absence of the quadruple hekat in the Middle Kingdom⁵⁹. As stressed earlier in this discussion, this measure is nowhere present in the account papyri (or elsewhere) of Dynasties 12 and 13. For this reason, I feel that Book II reflects the date of Rhind. That is to say, it is either a reworking of mathematical problems from an earlier time or else derived from a source different from that noted in the title page of the manuscript, and therefore different from Book I as well. It is true that no title or subheading appears immediately above or before the first problem of Book II; however, the break between these series of problems and those of Book I is so great as not to preclude this hypothesis. Moreover, where thematic separations occur in Book I, such as that between the opening divisions and problem 1, no massive and overt separation occurs. Hence, unless evidence for the quadruple hekat can be found in Dynasty 12 or 13, I prefer to divide Book I from II not only on the

⁵⁹ PSBA 14, 1892, 436.

basis of their topics but also from their original sources.

Nos. 41-43 are neatly written in the first block of this section⁶⁰. Each of the three occupies two registers and the last two present their calculation in columnar format. (Note the vertical $k\bar{i} n s\check{s}mt$, "the manner of the working out", at the beginning of these series of calculations in problems 42 and 43). In fact, all three examples separate the stages into the two registers in a neat and orderly fashion. The word employed for volume, $\check{s}3c$, carries with it the connotation of "empty" or in an extended sense, "vain", as Merenptah's Karnak Inscription indicates⁶¹. All three of these problems - and only these - deal with the volume of a circular cylinder, no. 44 turning to the volume of a square container. In problem 43 a correction was later made, the word tp being added in the middle of the second line. Peet has noted the difficulty of the problem as well as two minor slips: one in a fraction and the second by the addition of three words that make no real sense⁶². Without placing unwarranted emphasis on the last, I wish to stress this mistake. Following the phrase "this is the amount that will go into it" is $m ss h\check{k}3t$, "in grain, hekat measure". This makes no sense as the problem is moving to the determination of the amount of grain contained within a cylinder in quadruple hekat. This is, in fact, what the following words indicate: "in quadruple hekat". Perhaps an alteration from an original Dynasty 12 (or earlier) exemplar led to this switch and the copyist first failed to make his conversion into the newer system, only later doing so. In fact, there is a space larger than normal between the end of the first hekat ligature and preceding the fully-written quadruple hekat. Once again, speculation raises its troublesome head, and I would prefer not to read too much into this error, instead regarding the slip as one on the part of Rhind's scribe simply overlooking the use of the quadruple hekat and then quickly correcting his error. The original error was in any case not erased. However, as a result the final statement of the problem preceding the calculations in the lower register is somewhat crunched and extends a bit too far to the left. (It is in red ink).

⁶⁰ Chase-Bull-Manning, Photograph XVIII; Robins-Shute, Pl.14.

⁶¹ KRI IV 11.12; Peet, RMP, 80 (following Griffith). The idea of an empty vessel is self-evident. One might add Gunn, in: JEA 12, 1926, 131; Gardiner, AEO II, 212* on $\check{s}3\check{c}t$, the "garner" (cf. JEA 27, 1941, 24 note 2 - for an empty building later filled with grain?); and Alex I 362, II 369 ($\check{s}3\check{c}t$, the noun).

⁶² RMP 82-84 and especially page 82 note 1.

It is just possible that this was written later, since the margins of the next block to the left are aligned without this lengthy extension being taken into consideration; i.e., the right-hand margin of the next block is to the right of the final statement line of example 43⁶³

No. 44, which moves the volumes from circular cylinders to square containers, begins with the rubric *tp <n> nīš*, thereby paralleling the red heading *tp īrt* of no. 41 immediately to the left. As neither rubricised remark is found below at the beginning of the other similar problems in either block, it is readily evident that both subsections are self-contained units within Book II. In this case, 44 occupies two registers, as does 41, but nos. 45-47 are located within one register. The calculations for 44 are to be found in register two whereas the others are relatively simply located immediately underneath the opening statement of the problem. The next leaf occurs in the middle of problems 44-48, thereby supporting Griffith's old contention that two blocks (or pages) were frequently contained on one leaf⁶⁴. Part of problem 43 (in red) comes across the right-hand margin of the block containing nos. 44-47 so that the last is indented a bit to the left. No. 47 does not really fit those that are immediately above (44-46) and it is abbreviated as well. E. Peet maintained that "this example is nothing more than a statement of the fractions 1/10, 1/20, 1/30, etc. of a hundred-quadruple hekat". For him, "the opening words 'in a rectangular or circular cylinder' are unnecessary", and were possibly added by the copyist to connect this problem with those above⁶⁵. (A further connection between 47 and all of 41-46 is to be seen in the use of grain reckoning). In fact, as the layout is somewhat out of order at this point, there being a large blank in the final register and a brief problem (no. 48) having been written in an extremely condensed fashion quite to the left, I feel that the original set of examples ceased with no. 46, 47 being tacked on almost as an afterthought⁶⁶. *A fortiori*, observe that the latter deals with areas (as do nos. 49-55) and not volumes: a switch was clearly intended. Now, as the wording of no. 47 is different from that of 44-46 not only in the heading (*īr dđ n.k šš* [in red ink]...) but

⁶³ Chase-Manning-Bull, Photograph XIX; Robins-Shute, Pl. 15.

⁶⁴ For clarity: Chase-Bull-Manning, Photograph XIX and Robins-Shute, Pl. 14.

⁶⁵ RMP, 88.

⁶⁶ Chase-Bull-Manning, Photograph XIX; Robins-Shute, Pl. 15; cf. Peet, RMP, 88-89 for problem 48.

also in the entire problem, I would argue that the original set of volume problems concluded with 46, and that 47 may have been included by the copyist himself, independent of his source.

Whether or not no. 47 was to be found on the original is a moot point. Certainly, there are slips present throughout this section of the papyrus that indicate a less-than-perfect final copy. In the red inked heading to no. 44 the genitival n in $tp\ n\ n\dot{t}s$ is missing. Further confusion may be seen later on in the first line, and no. 45 lacks a hekat sign in its opening statement. No. 44 presents the heading $tp\ n\ s\dot{s}mt$ to the calculations in a column; no. 45 also has it, whereas no. 46 writes only $s\dot{s}mt.f$. Finally, no. 47, the "odd man out", contains the odd wording (and arrangement) that led Peet to believe that it originally was nothing more than a simple arithmetic problem⁶⁷.

No. 48, located in the sixth and last register of the block, is extremely abbreviated in content. Indeed, it presents a nice picture of a circle with radius 9 inscribed in a square as well as the calculations, but no opening statement occurs. And although it belongs to the following group of area problems (49-60), it bears all the earmarks of a hastily-added example located in an extended break (the blank under no. 47) which occurs neatly between the volume examples and those dealing with area that are next. Indeed, the Chase-Bull-Manning diagram is slightly misleading as no. 48 is placed within the block of examples 44-47 and does not run leftward to the margin⁶⁸. I wonder if it were a later addition, hastily written, perhaps by the master.

Examples 49-55, all occupying a single register, present a very orderly grouping and except for nos. 54 and 55, they all contain pictures of the geometric situation involved⁶⁹. The first four deal with pieces of land and although minor slips may be found, they all have standard openings (in red) followed by a picture of the problem and then the calculations⁷⁰. Depending upon the diagram, the latter figures may be to the left or else underneath the diagram. In example 53 we meet once more an incomplete problem, as the opening rubric is missing as well as further statements concerning the problem. I assume that the

⁶⁷ Peet, RMP, 88.

⁶⁸ Page xvi.

⁶⁹ Chase-Bull-Manning, Photographs XX-XXI; Robins-Shute, Pl. 16.

⁷⁰ See the various comments of Peet, RMP, 90-97 with those on the translation pages of Chase-Bull-Manning.

original lacked these words as well⁷¹. It is located to the left of 54 (not noted in the standard diagram of Chase-Bull-Manning) and I suspect was written after it as well. It has been labeled no. 53 simply because of its connection to example 52 and those immediately preceding. However, there is no doubt that it was written after no. 54 and placed within the fifth register to its left⁷². This would be simple to do as 53 is abbreviated, lacking the introductory heading and opening explanatory words. Once more the modern interpreter is faced with the choice of whether or not to speculate about the causes for this misalignment. In this case, I feel that 54 was inadvertently omitted by the scribe, quite possibly owing to the arrangement on his original. Then, after discovering his error, the copyist simply replaced the missing example to the left of 54. However, owing to limitations of space, he cut out much of the problem, leaving only the bare bones for us to work with. No. 54 itself, to the right, is a different problem, and one that perhaps did not need an explanatory diagram. As such, it parallels the final one (no. 55) in this group of examples. Both 54 and 55 deal with the division of a parcel of land into smaller units; both lack accompanying diagrams. The headings to them are identical, opening with the word "to subtract", *hbi* (in red ink). The former of the two is more abbreviated than the latter and neither occupies much of the register in which they are located. Needless to say, their right-hand margins are flush with the examples preceding: i.e., the layout remained the same as at the beginning of these problems. From the present setup it is clear that 55 could have fit into the large space to the left of 54 in the fifth register, but instead, no. 53 filled that blank.

In all, these series of area problems, although having thematic breaks, form a unity among themselves just as did the preceding examples covering volumes. Once more, they fill in one complete columnar block (excluding the anomalous no. 48), as, indeed, do the remaining examples of Book II (nos. 56-60).

With the six remaining problems of Book II we come to an effective break in Rhind⁷³. They are all basically parallel, set up in a well-

⁷¹ In particular, Peet, RMP, 95-96; Chase-Bull-Manning, Photograph XX; Robins-Shute Pl. 16.

⁷² NB: no. 54 is to the right of no. 53.

⁷³ Chase-Bull-Manning, Photographs XXI-XXII; Robins-Shute, Pl. 17.

designed fashion with the related diagrams of triangles (for pyramids in the first four examples) drawn at the extreme left. In fact, the block that they occupy begins on a new leaf. No. 60, only a bit different, deals with a *ḫwn* instead of a pyramid (*mr*)⁷⁴. In the first, the rubricised portion is the heading, as expected: *tp n nḥs*; the later ones write only *mr* ("pyramid" or *ḫwn* in red ink). A slip may have occurred in this case; the calculations appear between the text and the diagram to the left. No. 57 writes its final line in a somewhat cramped fashion by placing it to the middle of the register block, quite possibly as the scribe was afraid that he might run too far to the left and thereby constrict the space for his plan of the accompanying pyramid. No. 58 is likewise of a set arrangement, although in this case the word for pyramid, *mr*, is written with the two alphabetic signs separate from each other, a full hieratic *m* (not an abbreviated one) preceding the *r*. Also interesting is the use of the cursive ligature *pn* - "this", first noticed by Gunn: this also occurs in problems 56 and 67 on the verso⁷⁵. In this case limitations of space appear to have been the cause. The fourth register includes nos. 59 and 59b; both are related as they deal with the same situation. Hence, only one diagram was needed. The final words of 59 are corrupt, and possibly the original text was as well; as it stands this example also shows other problems. 59b is more compressed as it had to be placed in a limited space to the left of its companion. The final case, which deals with a column, has an interesting scribal slip wherein part of the result statement was incorrectly inserted into the operations portion⁷⁶. The diagram to the left is a bit different from those pyramids preceding (the base is drawn with one instead of two lines), and it may have been an intentional way of marking a difference between it and the others in a very schematic fashion. Peet has made the further point that the entire wording appears confused (cf. his discussion of the word *stwtj*)⁷⁷.

Book II is now finished. The problems that comprise it are in no way connected with the series of quite different problems that are to be

⁷⁴ Peet, RMP, 101 and text of "Problem 60" in Chase-Bull-Manning.

⁷⁵ Peet, RMP, 101 with note 3 to "Problem 56", note 2 to "Problem 58" and note 8 to "Problem 67" in Chase-Bull-Manning; cf. Gunn, in: JEA 12, 1926, 134.

⁷⁶ Peet, RMP, 101.

⁷⁷ Ibid.

found on the recto. Indeed, the relatively straight-forward arrangement and internal thematic divisions cease at this point. In fact, the entire set of cases on the recto follow a progressively more detailed and easy-to-determine system. From calculating tables of a simple but necessary kind the papyrus moves into more realistic problems that the student would eventually have to face in his career. The advanced arithmetical problems conclude the first Book. The second deals with realistic two and three dimensional problems, interestingly commencing with the latter (volumes) before turning to the former (areas). All cases of Book II are concrete, as the basic arithmetic is assumed. I feel that the second portion of the recto could have been presented to an advanced class of students after they had mastered the first examples, and probably the tables at the beginning had to be memorized and copied down by the apprentices in their own notebooks.

Leaving off for the moment the continuation onto the verso (see Part D below), I would only conclude by noting the strong disparity between Books I and II; the fact that the gap between them is large enough by itself to warrant speculation that the two books derive from the same source; and the quadruple hekats in the latter book. The entire side was ultimately covered (excluding the gaps), as befits a master tool for teaching and review, whereas the verso presents a quite different appearance.

D. BOOK III

Peet's heading for this section of the Rhind papyrus is "miscellaneous problems"⁷⁸. In this he was following other scholars such as Griffith, who likewise observed the heterogeneous nature of the verso. Although some organization can be found within most of this section of the work, the back of the papyrus exhibits the complexity all too common with Egyptian hieratic documents: viz., the verso played a secondary role in the final production of manuscripts, especially literary (or scientific) works. In particular, later additions were apt to be written on this side (cf. the well-known addition to P. Ebers), and the strict layout of the recto was often abandoned. The case of Rhind is a useful one to review as, more than most, its somewhat incoherent

⁷⁸ Ibid., 103 with Griffith, in: PSBA 16, 1894, 238.

arrangement is worthy of a detailed analysis.

First and perhaps foremost, is the addition of no. 61, a multiplication table of fractions, as well as 61b, a simple arithmetical problem similar to those in the opening of the recto⁷⁹. Both are located to the right of the double vertical lines and with Peet, among others, I feel that they were added later as useful case studies to help the reader (or teacher).

The text proper originally began with no. 62⁸⁰. It and the following two problems fill up one block. The first is a famous one as it deals with exchanges involving (silver) "pieces". Significantly, the relationship of one deben of weight to 12 "pieces" can also be found at the end of the 18th Dynasty, a point that Gardiner stressed in his important breakthrough of the Kahun Papyri, which were first edited by Griffith. In the Rhind example the presupposed standard was gold, and Černý has observed that "as early as the Middle Kingdom" gold had taken prominence over silver⁸¹. Unfortunately, all that this conclusion allows us is to date the problem internally to the 12th Dynasty rather than earlier. By the time the Kahun Papyri were written, the exchange was with silver, not gold, although in a mid 18th Dynasty Cairo papyrus Peet noted that the gold "piece" was probably regarded as more fundamental than the silver "piece". Again, the Rhind example cannot provide a definitive criterion concerning its original date. And although giving a gold:silver:lead ratio of 2:1:1/2, case no. 62 provides little other information helpful to our researches. Indeed, one of the spellings of the "piece" as $\check{s}<n>^c ty$ is not very useful as it later turns up at the beginning of Dynasty 18, and so no *terminus a quo* can be made.

Internally, cases 62-64 present interesting anomalies⁸². The first has a corrected error of the spelling of $\underline{d}h ty$, "lead". The example is written in the first three registers, the first always being of a

⁷⁹ Peet, RMP, 103-04; Chase-Bull-Manning, Photograph XXIII; Robins-Shute, Pl. 18.

⁸⁰ Peet, RMP, 104-07; Gardiner, in: ZAS 43, 1906, 45-47; Gunn, in: JEA 12, 1926, 135 (on the writing of $\check{s}^c ty$); Černý, Cahiers d'histoire mondiale 1, 1953/54, 905; Peet, "The Unit of Value $\check{s}^c ty$ in Papyrus Bulaq 11", in Mém. Maspero I, 185-99; Helck, in: AoF 15, 1988, 7 with note 6; and Janssen's comments in Commodity Prices from the Ramessid Period, 102-05. The most recent studies on $\check{s}^c ty$ are: Helck, in: AoF 15, 1988, 6-8 and Janssen, *ibid.*, 13.

⁸¹ "The Unit of Value $\check{s}^c ty$ ", 196-97 in particular.

⁸² This is best seen in Robins-Shute, Pl. 18.

lesser height than that of the five below, as on the recto. The opening words, *tp n irt krft*, are in red ink and the three portions of the problem are written in separate registers. In the next one, the fourth, patches were added after a break occurred, and some partially broken signs were repaired⁸³. This example covers registers four and five and deals with shares of loaves divided among four men. As such, it is distinct from the previous case but similar in outlook to the ones immediately following. The calculations, introduced by the rubricised words *irt mī hpr*, are placed separately in the fifth register and as a result the spacing is more than reasonable. No. 64, which occupies the final register in the block, has four lines that overrun the original left-hand margin and a notation, *ch^o*, has been placed further to the left in the first line; the black ink is fainter than normal. In fact, the hieratic is cramped in an effort to fit it within the one register. The subject of the problem covers distribution of grain (barley) and is connected by this theme to most of the opening cases on the recto.

With no. 65 we turn to the second block on the verso⁸⁴. Again, the opening rubric employs the common "case/example of ..." (*tp n ...*), one of the marks that these problems, although diverse in nature, do possess an underlying unity. In essence, they are specific problems of an advanced nature covering common situations that the apprentice would have to become aware of before he embarked on his career. Each of these opening problems on the verso involves fractional shares, differences, and involved calculations. Even no. 67, which covers the produce of a herdsman, is in essence nothing more than finding an integer given a result (as a proportion, or a fraction of a fraction of that unknown). In no. 68 we meet for the first time the "great quadruple hekat", whose importance has previously been stressed in this discussion. Finally, with no. 70, the right side margin is indented somewhat owing to the "overflow" from case 64.

Peet and others noted a decided switch in theme by the time problem 69 is met⁸⁵. At this point, the topic is solely concerned with bread and beer exchanges in combinations with their *psws*. This subsection

⁸³ Ibid.; cf. Peet, RMP, 107. Was the patch added in Thebes?

⁸⁴ Chase-Bull-Manning, Photographs XXIV-XXV; Robins-Shute, Pls. 19-20.

⁸⁵ RMP, 112-14.

of Rhind's verso parallels a group in the Moscow Mathematical Papyrus to such a degree that I believe them to be both derived (though independently) from a major treatise solely concerned with bread and beer problems⁸⁶. In the latter, the relationships are actually more difficult to work out owing to the presence of various types of grain (barley, emmer) as well as dates. Somewhat surprisingly considering the value that Rhind must have possessed in its day, the cases on the verso are easier and more simplified. By the time no. 80 is reached, the neat arrangement of the papyrus is finally lost; this will be covered below.

Returning to the block of problems commencing with 65, we can first observe that they all occupy one register. Where the calculations plus explanations are quite long, the scribe has chosen to continue on to the left, employing separate columns to draw up his calculations, checks, and "proofs". Hence, nos. 68-70 extend well along into the papyrus, the first two being of the greatest length. Owing to this setup, there was enough space in the first three registers to write three more problems (by now the bread and beer examples have been reached), and so the layout is a bit irregular at this point: 71-73 are flush with 65-67 and above 68-70⁸⁷.

In cases 65-68, which conclude the first group of somewhat varied day-to-day real problems, a few slips may be seen (cf. no. 66 with the added 10 hekat). The writing is tighter and so the cursive *pn* could be written (no. 67). More interesting is the hieratic writing of six in case 68. As Bull noted in his remarks to the transcription, it resembles the later abstract integer 6 but can be found in problem 84 as well as in the Illahun papyri⁸⁸. The presence of a patch in the same problem ought to be stressed; it was later overwritten, and useful remarks concerning this may be found in the Chase-Bull-Manning edition.

With no. 69 the second subsection of Book III begins. Here, the problems are quite narrow in orientation. Perhaps the stress on single

⁸⁶ In SAK 15, 1988, I argued against some of the statements of Nims in: JEA 44, 1958, 56-65.

⁸⁷ It is useful to see the original at this point in Chase-Bull-Manning, Photographs XXIV-XXV; Robin-Shute, Pls. 19-20.

⁸⁸ See note 10 to "Problem 68" in Chase-Bull-Manning.

hekats is important for its dating, and I feel that a good case may be made for its original to be dated to the Middle Kingdom (unlike, for example, no. 68). The first two problems occupy the final two registers of the block, with enough space occurring at the bottom for an additional case (no. 79: see below) to be later added simply to fill in the sixth register. The next three, 71-73, occupy the large space in the first three registers next to nos. 65-67, and the numeration of Peet, Griffith, et al., appears to be valid. Indeed, no. 73 parallels the following case in presenting a problem of exchange connecting loaves of differing *psws*.

On another occasion I have had reason to stress the uniqueness of problem 71⁸⁹. It is a simple case of the dilution of beer (with an assumed *psw*) with water, rather than the constant use of dates as in the Moscow Papyrus. As such, it ought to be the abstract paradigmatic case for a series of concrete examples concerned with *bš3-bnr*, although it is not followed by any of them. The uniqueness of the problem is highlighted even more by its inclusion at this point. The previous examples (68-70) cover loaves of bread, as do those that follow (72-75). Only in 76 do beers reappear and even then, this problem and the two succeeding ones cover exchanges between beers and breads, both having various *psws*. In all, I feel that no. 71 stands out by itself, lacking any complementary real examples which would have been given by the master teacher to his student. It might even be argued that no. 71 represents a later version of what the Moscow Mathematical Papyrus concerns itself with. That is to say, it reveals a later stage during which the *bš3-bnr* problems had become less important. Certainly, by Dynasty 18, the offering calendars avoid such mixtures of liquids whereas in the Middle Kingdom (cf. P. Bulaq 18 and the Illahun material), combinations of *bš3* and *bnr* were present.

Be that as it may, the following cases return to bread exchanges. A slip is to be found in no. 73, no. 76 contains a mistake, and an omission was later added in over a line in no. 78⁹⁰. With 74-78 a new

⁸⁹ SAK 15, 1988.

⁹⁰ For convenience see the comments in Chase-Bull-Manning: note 7 to "Problem 73", note 7 to "Problem 76", and note 1 to "Problem 78". The text is in Chase-Bull-Manning, Photographs XXV-XXVII and Robins-Shute, Pls. 20-21.

block is formed, and I feel that the original arrangement is still clear (see Diagram 1). However, the last of these effectively concludes this series of advanced problems. All of them are aligned to the right and only the last is short. 77-78 open with the expected rubricised *tp n* whereas 74-76 employ (in red) *ky*, "another <case of>". The first subgroup therefore refers back to the preceding one, which deals with the exchanging of loaves of one strength (or amount of grain) with others having a different *psw* (nos. 72-73). Nos. 77-78, as they cover bread and beer exchanges, therefore have a different heading. The problems in this last group occupy one register apiece, and no. 77 has ample space below it, testifying to the deliberate intent of the copyist to restrict his problems to a neat and orderly arrangement.

Problem 79, perhaps misnumbered, occurs in the sixth and final register of the preceding block⁹¹. It was placed there simply to fill up the blank caused by the brevity of problem 70, although it overruns the lower margin to a degree. Truly, it is out of place! This example is the famous one to which the rubric "The St. Ives Case" seems quite appropriate. The black ink employed is somewhat lighter than in the surrounding examples, and perhaps more significant, no red ink occurs. In fact, there is no heading. Griffith commended the problem to the Folklore Society, in a sentence that was not at all flippant⁹². In essence, this problem is independent of those surrounding it. It is out-of-place in theme as well as mathematical calculation, being in essence a classic case of a nested odd integer (7): 7 times 7 times etc. I suspect that whatever the original intent may have been on the part of the copyist, be it whimsy or otherwise (with the possibility that it was added later by a different man), it seems best to regard it as a popular mathematical puzzle, well-known to Egyptians of the day, and its inclusion at this point lends to the final version of the verso a symmetry and completion previously lacking.

Problems 80 and 81 belong together by virtue of their theme: both are in essence the subdivision of the grain measure in *henus*⁹³. The first,

⁹¹ Peet, RMP, 121-22; Chase-Bull-Manning, Photographs XXV-XXVI, and Robins-Shute, Pl. 20.

⁹² PSBA 16, 1894, 242.

⁹³ Chase-Bull-Manning, Photographs XXVI-XXVII and Robins-Shute, Pls. 20-21.

rather short, occupies the left-hand side of the fifth register in the block so far discussed (see Diagram 1). Useful equations are made between one hekat and the various dimidiated fractions down to the final $1/64$ of the Horus-eye notation and their equivalent values in henu. It is nothing more than a table for future reference. No. 81, on the other hand, presents a more detailed and lengthier conversion of fractional hekats (commencing with $1/2$) also into henus, although in this case there are numerous combinations of fractions. Again, it is nothing more than a very detailed and helpful chart for actual computational purposes. The opening rubricised words, "another reckoning of the henu measure", overtly indicates that no. 81 is to follow no. 80. There is a columnar organization to the former, and some misuses of red and black ink were already observed by Peet⁹⁴. In addition, the lower margin has once more been disregarded by the scribe, as he purposely avoided moving into another block. More interesting is the extreme length. It ends up overrunning the left-hand margin of the block formed by nos. 74-77 (first four registers) and 78, 80 (fifth register, the latter to the left of the former). In fact, example 81 proceeds further left than example 83, which overlies it, and ends up somewhat alone right next to the large blank forming the end of the original series of problems on the verso, as if the copyist knew that he was almost at the end of his original text.

Nos. 82 and 82b, written in the first three registers of the following block, are not separated from each other⁹⁵. Griffith was the first to point out that no. 82b, written in the third to fifth lines of the third register, is a problem different from its companion⁹⁶. Both concern themselves with feed (with the hekats, double-hekats, and various subdivisions) in a poultry farm. There are enough errors to suspect a lapse in the original or at least poor copying by the scribe. In fact, the accuracy of these final problems of Rhind begins to slip to such a degree that I wonder if the examples in the original were selectively chosen and hastily composed. Further on, problem 83 (registers four and five) runs more to the left than nos. 82 and 82b, further testifying to the scribe's avoidance of a neat symmetrical block form-

⁹⁴ RMP, 124.

⁹⁵ Chase-Bull-Manning, Photograph XXVII and Robins-Shute, Pls. 21-22.

⁹⁶ PSBA 16, 1894, 224; cf. Gunn's comments in JEA 12, 1926, 136. On the word *h3(t)-mw*: Caminos, Chronicle of Prince Osorkon, Rome, 1958, 68 note k; Meeks, in: RdE 28, 1976, 92-95; and ALex I 225/227.

ation. Clearly, reaching the concluding problems, he did not observe the necessities of a neat arrangement. The topic of this example is again related to feeding and, of course, the hekat measurements are once more employed. After completing the first half of the problem, which deals with fattening geese (as in the previous two cases), there is a useful list of how much various birds eat. The latter is not a mathematical problem but simply a handy list of costs (in fractions of grain). It is located in a separate column (lines 6-10: register five) of the case, and I feel that it ought to be separate from the problem itself, if only on the basis of its differing content.

The last problem of Rhind (no. 84) is the most garbled, as if the scribe had tired of his work (or the original was even more faulty), and quickly completed his task⁹⁷. Located to the left of nos. 82 and 82b, this case of feeding oxen (instead of fowl) runs into the third register by two lines; there is no symmetry with the other examples. It may be useful to remark that the horizontal guidelines that form the various registers and margins cease (in dots) at this point: i.e., problem 84 concludes on the left with the end of these lines. Can we argue that the final length of Rhind on the verso was assumed from the onset of the work and that the copyist had to fit into it as many problems as possible, and so the concluding examples became haphazardly set up and poorly arranged? Or, perhaps more correctly, was it the case that the copyist approximated the length of the verso to be used but that he was off by a little. As a result, the concluding problems were crunched (cf. 82 and 82b; 83), and many slips occurred. This last case is the most poorly presented as it contains too many mathematical errors to overlook. However, it can equally be said that the possibility of a corrupt or badly damaged (or faint?) original may have been the cause of the garbled account of no. 84, and some of those immediately preceding as well.

Problem 84 ends the original layout of papyrus Rhind and it is possible that the end of the original text occurred at this point⁹⁸. The presence

⁹⁷ Chase-Bull-Manning, Photographs XXVII-XXVIII and Robins-Shute, Pl. 22.

⁹⁸ This is best seen on Robins-Shute, Pl. 22; cf. Pls. 23-24. (Pl. 23 top = no. 85; Pl. 23 bottom = no. 87; and Pl. 24 = no. 86). The center division lines appear to stop at the end of the mathematical portion of Rhind. In Robins-Shute, Pl. 23 (top), two registers can be seen but the text does not coincide with them.

of dots at the end of the horizontal margins and register divisions points to the reasonable supposition that the copyist knew approximately when he was to conclude this work. Griffith observed that the writing ceased at the conclusion of the fourth leaf, and thereby the original intent of the scribe is even more revealed⁹⁹. Certainly, the spacing below in registers four and five (three was partially used by the last two lines of no. 84) is not large enough to hold another problem, and to all intents the end of the text seems well designed. Whether or not the last problem belonged to the original is unknown, although its garbled condition may predicate a source different from those cases immediately preceding. Indeed, one may wonder whether Book III derived from the same mathematical treatise as Book II; however, as with most of Rhind, this remains moot.

E. THE ADDITIONS OF THE VERSO

After the papyrus had been completed, and undoubtedly after some use as a teaching manual, later remarks were written on the verso in the great blank following problem 84. The first case after this example has been numbered 85 by previous scholars, and I will follow that designation here, although believing that it is in fact later than that and should be at least no. 86. By itself, 85 is a curious jotting¹⁰⁰. Upside down, in a different (and thicker) hand than that of the original scribe, it presents an early case of cryptographic writing. Gunn, in his review of Peet, was the first to attempt a concise evaluation of the meaning, and he observed the presence of such writing from Dynasty 18 on, citing examples from Theban tombs, as well as other monuments from that capital¹⁰¹. 85 is to be found on P. BM 10058 and was later damaged by the application of a patch, probably ancient, which indicates that this portion of the papyrus was not considered significant by the repairer. Following Gunn, I feel that the presence of cryptography at this point ought to predicate a date within Dynasty 18, and the eventual location of Rhind at Thebes just may supply some support for this supposition. After all, it is from that city that we

⁹⁹ PSBA 13, 1891, 330.

¹⁰⁰ Chase-Bull-Manning, Photograph XXIX with Robins-Shute, Pl. 23 (top).

¹⁰¹ JEA 12, 1926, 136-37 with the older literature and dating of this portion of Rhind. Most recently, see Hornung-Staehelin, *Skarabäen und andere Amulette aus Basel Sammlungen*, Mainz 1986, 173-80; Hornung, in: JSSEA 13, 1983, 173-80; cf. Grapow, in: ZÄS 72, 1936, 23-29 with Rodrigo, in: JSSEA 15, 1985, 68-70 (Dynasty 19 historical text of Ramesses II).

known the most about this so-called enigmatic writing, and such texts are dated to the New Kingdom and not earlier. Moreover, the fact that 85 was written upside down, unlike 86, shows that it was not meant to be read by someone conversant with the contents of Rhind but probably intended as an afterthought, perhaps by a later user or owner.

With no. 87, located on P.B.M. 10057 roughly in the center, Rhind presents the famous and highly-debated jottings concerning the taking of Avaris by Ahmose¹⁰². I feel that it was added to the middle of the verso, and rightside up, so to speak, soon before the entire roll was transported to Thebes from the north. Möller, in fact, preferred to see the hand as probably identical to the scribe of the insert no. 86, and one can note the more cramped style and more cursive use of hieratic at this juncture¹⁰³. The brief remarks provide not merely a *terminus a quo* for the presence of Rhind later than year 33 of the Hyksos ruler Apophis, they also indicate that a major historical event was purposively written down on a mathematical tractate, itself being of high importance and value. Soon after, Rhind was, I believe, transported back by someone in the victorious Theban army to the new capital and later used there as a treatise, only to have a further addition entered (no. 87). No other specifically dated historical remarks occur on the verso after this date, which is itself ambiguous. I feel that the regnal dates do not refer to the reign of Ahmose but rather to that of the last Hyksos ruler in Egypt, a position that I am well aware is open to question; however, the historical event is at least clear: viz., the end of Hyksos control in the eastern Delta (Heliopolis and Sile are noted as having fallen)¹⁰⁴. If we follow Möller, then the possessor of Rhind at that time felt these major events worthy of a remark on one of his prized treasures. As for the order and arrangement of these brief remarks, note the use of red ink for the dates and the division into three columns, the first having nothing below its single line, almost as if other, more detailed remarks (as in the remaining columns), might have been added later; the days are included in the second and third columns.

¹⁰² Chase-Bull-Manning, Photograph XXX and Robins-Shute, Pl. 23 (bottom).

¹⁰³ Paläographie I, 18 ("etwas spätere Zeit"); Peet, RMP, 129 (around same time as main text of Rhind) with note 1 page 130 (beginning of Dynasty 18?); for Möller's comments on Dynasty 18 hieratic: ZAS 56, 1920, 34-43 (now dated).

¹⁰⁴ Helck, in: GM 19, 1976, 33-34 and Vandersleyen, Les Guerres d'Amosis, 34-40.

No. 86, on the other hand, was written upside down, as was no. 85¹⁰⁵. It was made up of three strips later pasted onto the verso of the papyrus close to the left in an effort to strengthen the papyrus roll. It is an account and quite interesting, as we can see the presence of the quadruple hekat among the fragmentary notations. Furthermore, they were joined to the rear of Rhind out of order, thereby indicating their insignificance vis-à-vis their original content. Peet, who recognized the cursive nature of the script, left the date of these pieces open, although he noted Griffith's conclusion that the scribe was identical to the copyist of Rhind itself¹⁰⁶. The name Kamose in line x+2 of the account provides a vague terminus around the 17th-early 18th Dynasty, although the presence of the city *Hbntw*, probably that located in the 14th nome of Upper Egypt, may seem too distant from the Hyksos domain to many modern researchers. Various produce are converted into silver, a point upon which Griffith thought useful to remark, for if his dating is correct, then "the use of a silver unit in Egypt for estimating value is thus taken back to the age of the Hyksos"¹⁰⁷. The writing of the grain values is of some use as a dating criterion: the hekat signs as well as that for the quadruple hekat certainly predate mid-Dynasty 18, but to what epoch the text specifically belongs cannot be absolutely determined. Still, the older use of hekat measuring is of some value. One might note the use of red ink for *bd̄t* (emmer) and black for barley (*l̄t*) and the fact that the first line is actually the beginning of the original text: it commences with the end of a king's name (now lost) and then with the heading "Account of the food in *Hbnty*". Prices are given, but the pitiful fragments leave too little for any precise determination of grain values. In all, I would date the three fragments to a period of time not far removed from the original text of Rhind, and perhaps Griffith was right in adhering to the time frame of late Hyksos. As a caveat, it ought to be stressed that the date of these fragments need not coincide with the time in which they were employed as backing. As a discarded papyrus, its value would no longer be great and its mundane

¹⁰⁵ Chase-Bull-Manning, Photograph XXXI and Robins-Shute, Pl. 24.

¹⁰⁶ Peet, RMP, 128; Griffith, in: PSBA 16, 1894, 248 for his now dated comments on the use of silver; I ignore the location of *Hbnty* (in the 14th nome of Upper Egypt: Peet, 128 and note 2, who follows Brugsch, Dictionnaire Géographique, Leipzig 1879, 469).

¹⁰⁷ Ibid.

reuse could have taken place some years after it was first drawn up. I tend to feel that Peet's more cautious outlook is correct: no. 87 is "little if at all later in date" than Rhind itself.

F. CONCLUSION

This somewhat discursive yet detailed survey of the internal characteristics of Papyrus Rhind is helpful in showing the organization of the composition as well as the intent of the copyist. I have preferred to eliminate any discussion of the mathematics involved, leaving such work to the more competent scholars whose studies of the subject matter of this composition are extremely worthwhile. The internal history of the treatise, including the later additions, sheds a small yet significant light upon Rhind's development, and its final resting place in Thebes, miles away from its original location, provides an even more intriguing reason why I felt that this analysis was necessary. Oddly enough, Rhind does not figure in the worthwhile survey of Černý, and Caminos' important recent evaluation of reused papyri is ancillary to this study. In essence, Rhind reveals a well-worked development on the part of the copyist, whose careful spacing and beautiful hand provide us today with an excellent exemplar of a first class hieratic document, the value of which is augmented by its excellent state of preservation. Complete as well, the work has been employed as a basis of palaeography, a study of Egyptian mathematics, and an analysis of Middle Kingdom papyri. In fact, although dating to the later Hyksos period, and - useful to remember - from Lower Egypt, this papyrus reveals its close connection to the Middle Kingdom, as its dimensions overtly testify. But for the purposes of this study, I have deferred any detailed analysis of this aspect of the text, preferring instead to present Rhind as a composite text, and thereby viewing it from a vantage point often considered to be mundane. As a historical document, and just by itself, this mathematical work can stand on its own. Despite or perhaps owing to the complicated internal make-up, Rhind is one of the most important hieratic works of Pharaonic Egypt, and it looks back to the days of Dynasty 12 rather than prefiguring those of the next great phase of Egyptian civilization.